

$$\begin{aligned}
&= \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a} \right)^{3/2} e^{-Zr/a} \\
&= \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a} \right)^{3/2} \left(2 - \frac{Zr}{a} \right) e^{-Zr/2a} \\
&= \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a} \right)^{5/2} r e^{-Zr/2a} \cos \theta \\
&= \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a} \right)^{5/2} r e^{-Zr/2a} \sin \theta \cos \varphi \\
&= \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a} \right)^{5/2} r e^{-Zr/2a} \sin \theta \sin \varphi \\
&= \frac{1}{81\sqrt{3\pi}} \left(\frac{Z}{a} \right)^{3/2} \left(27 - 18 \frac{Zr}{a} + 2 \frac{Z^2 r^2}{a^2} \right) r e^{-Zr/3a} \\
&= \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a} \right)^{5/2} \left(6 - \frac{Zr}{a} \right) r e^{-Zr/3a} \cos \theta \\
&= \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a} \right)^{5/2} \left(6 - \frac{Zr}{a} \right) r e^{-Zr/3a} \sin \theta \cos \varphi \\
&= \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a} \right)^{5/2} \left(6 - \frac{Zr}{a} \right) r e^{-Zr/3a} \sin \theta \sin \varphi \\
&= \frac{1}{81\sqrt{6\pi}} \left(\frac{Z}{a} \right)^{7/2} \left(\frac{Z}{a} \right) r^2 e^{-Zr/3a} (3 \cos^2 \theta - 1) \\
&= \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin \theta \cos \theta \cos \varphi \\
&= \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin \theta \cos \theta \sin \varphi \\
&= \frac{\sqrt{2}}{81\sqrt{2\pi}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin^2 \theta \cos 2\varphi \\
&= \frac{\sqrt{2}}{81\sqrt{2\pi}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin^2 \theta \sin 2\varphi
\end{aligned}$$